Instructions

1) In your folder you will find out the following:
   a) Answer sheets
   b) Draft sheets
   c) The envelope with the subjects in English and the translated version of them in your mother tongue;
2) The solutions of the problems will be written down only on the answer sheets you receive on your desk. **PLEASE WRITE ONLY ON THE PRINTED SIDE OF THE PAPER SHEET. DON'T USE THE REVERSE SIDE**. The evaluator will not take into account what is written on the reverse of the answer sheet.
3) The draft sheets is for your own use to try calculation, write some numbers etc. BEWARE: These papers are not taken into account in evaluation, at the end of the test they will be collected separately . Everything you consider as part of the solutions have to be written on the answer sheets.
4) Each problem have to be started on a new distinct answer sheet.
5) On each answer sheet please fill in the designated boxes as follows:
   a) In PROBLEM NO. box write down only the number of the problem: i.e. 1 – 3. Each sheet containing the solutions of a certain problem, should have in the box the number of the problem;
   b) In Student ID – fill in your ID you will find on your envelope, consisted of 3 leters and 2 digits.
   c) In page no. box you will fill in the number of page, starting from 1. We advise you to fill this boxes after you finish the test
6) We don't understand your language, but the mathematic language is universal, so use as more relationships as you think that your solution will be better understand by the evaluator. If you want to explain in words we kindly ask you to use short English propositions.
7) Use the pen you find out on the desk. It is advisable to use a pencil for the sketches.
8) At the end of the test:
   a) Don't forget to put in order your papers;
   b) Put the answer sheets in the folder 1. Please verify that all the pages contain your ID, correct numbering of the problems and all pages are in the right order and numbered. This is an advantage of ease of understanding your solutions.
   c) Verify with the assistant the correct number of answer sheets used fill in this number on the cover of the folder and sign.
   d) Put the draft papers in the designated folder, Put the test papers back in the envelope.
   e) Go to swim

GOOD LUCK !

Problem 1 Black Hole in Milky Way

By observational facts, the scientists admit presence of a black-hole at the center of Milky Way.
At the center of Milky Way, a hypothetical black-hole (Sagittarius A*) is located. A star S* is orbiting the black-hole SA*.

In the table 1 the following data is presented: the date and the angular position coordinates $(\alpha;\beta)$ of the star S* at different moments of the observation. The coordinates represent the angular distances of the projection or the star S* in the coordinates system (U, W), centered on the SA* (see figure 1).
An angular distance of $\varphi = 1\ \text{arcsec}$ corresponds to linear distance in the plane of the sky $d = 41$ light days, therefore to a scale $S_0 = \dfrac{d}{\varphi} = 41\dfrac{\text{light day}}{\text{arcsec}}$.

	Date (year)	α(arcsec)	β(arcsec)
1	1995.222	0.117	- 0.166
2	1997.526	0.097	- 0.189
3	1998.326	0.087	- 0.192
4	1999.041	0.077	- 0.193
5	2000.414	0.052	- 0.183
6	2001.169	0.036	- 0.167
7	2002.831	- 0.000	- 0.120
8	2003.584	- 0.016	- 0.083
9	2004.165	- 0.026	- 0.041
10	2004.585	- 0.017	0.008
11	2004.655	- 0.004	0.014
12	2004.734	0.008	0.017
13	2004.839	0.021	0.012
14	2004.936	0.037	0.009
15	2005.503	0.072	- 0.024
16	2006.041	0.088	- 0.050
17	2007.060	0.108	- 0.091

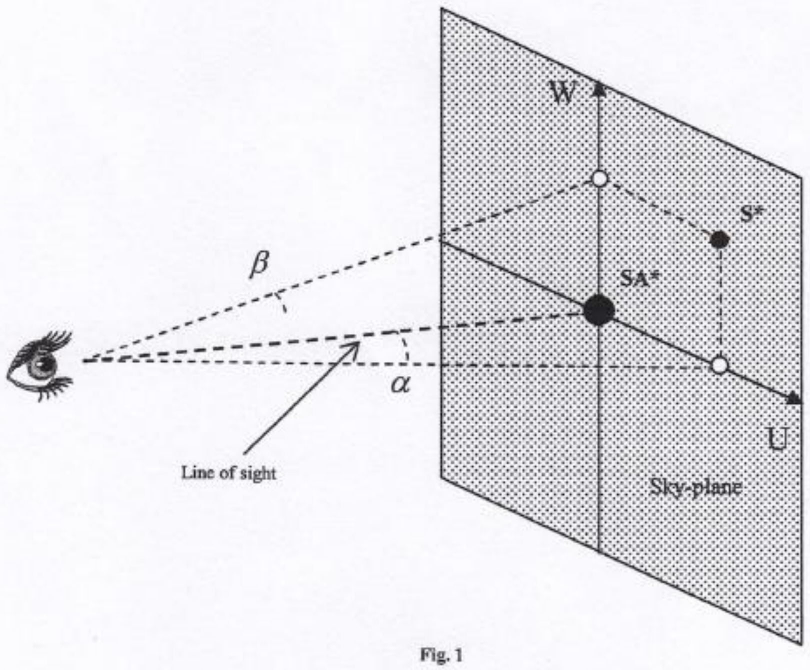


Fig. 1

By using the information provided your tasks are:

a) Plot the projection of the trajectory of the star S^* in the plane P (see figure 2). This plane is close to the observer. In this plane, $\varphi = 1\ \text{arcsec}$ corresponds to a linear distance d_0= 1200 mm therefore the scale is $S = \frac{d_0}{\varphi} = 1200 \frac{mm}{\text{arcsec}}$. You have to use the millimeter graph paper, carbon copy sheet of paper and the transparent sheets for an accurate plot.

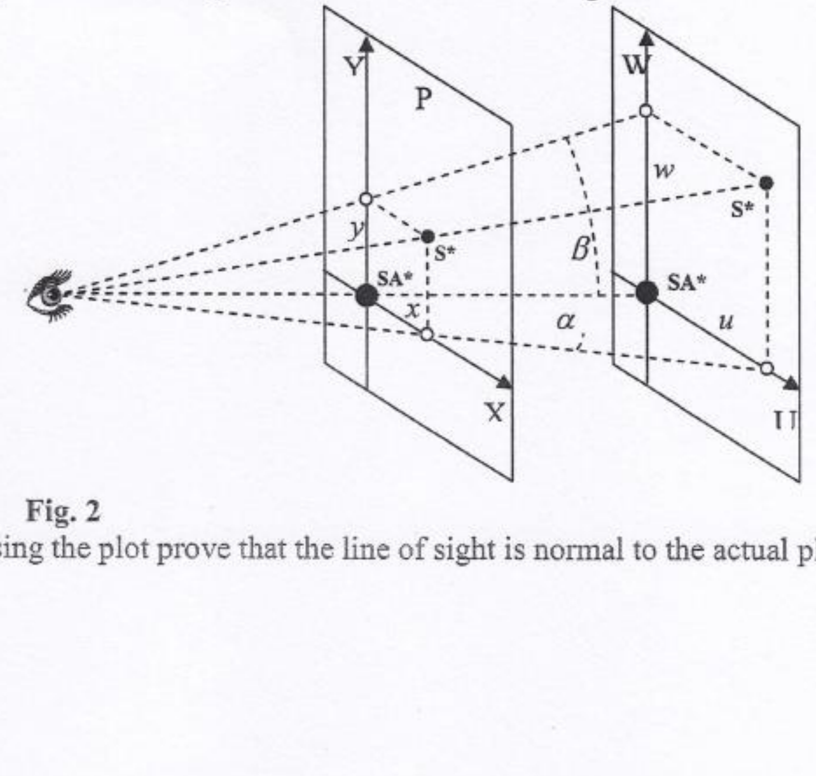


Fig. 2

b) By using the plot prove that the line of sight is normal to the actual plane of the orbit

c) Using your plot find out following elements of the real orbit of star S* around the black hole SA*:

I. $a-$semi-major axis (in light days units) ; $b-$small semi-minor axis in (in light days units); $e-$eccentricity;

II. $r_{min}-$the minimum distance between S* and SA*(in light days units); $r_{max}-$the maximum distance between S* and SA* (in light days units);

III. The distance from the observer to the S*;

IV. the orbiting period of star S* around SA* (obtain the best possible result by taking as many measurements as possible and by taking their arithmetic mean);

V. the total mass of the system "SA* - S*".
Presenting the intermediate and final data in tables is recommended for an accurate evaluation.
$G = 6.67 \cdot 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$

Problem 2Thermodynamic test

An hypotetical shuttle is launched to investigate the atmosphere (100% CO_2)of two extrasolar planets P_1 and P_2 . The atmosphere is in static thermodynamic equilibrium. When the shuttle is near each planet, a radio probe is launched toward respective planet, invertical direction (in the direction of the planet`sradius). When the radio probe reaches constant velocity, it starts sending values of the pressure of the atmosphere. In Fig. 3.1 is plotted the atmospheric pressure values (in arbitrary units) as function of the time of descent for the planet P_1. When the probe touches the surface of planet P_1 it sends the value of the temperature $T_0 = 700\ \text{K}$ and the value of the gravitational acceleration $g_0 = 10\ \text{ms}^{-2}$.

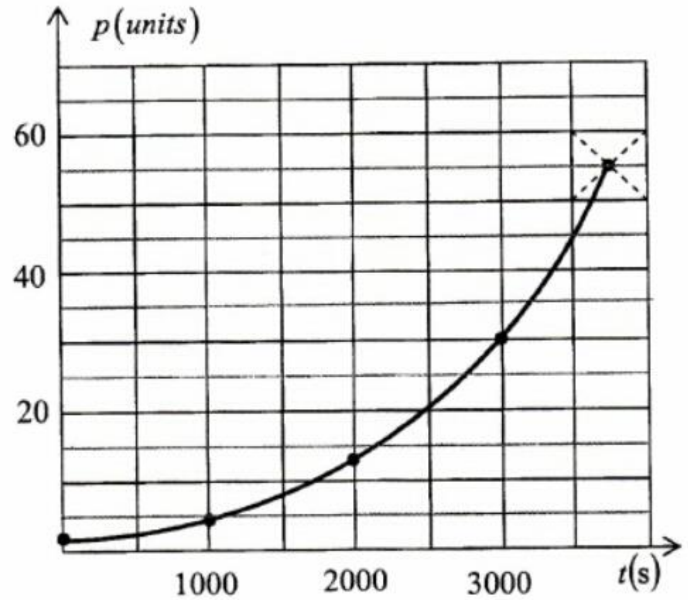


Fig. 3.1.

The gravitational acceleration on each planet is assumed to be constant during uniform descent of the radio probes.

a) Find the altitude h_0 from where the radio probe R_1 starts the uniform descent and thus starts the transmitting information.

b) Find the temperature of planet P_1 at the altitude $h = 39.6\,\text{km}$. You know: The universal gas constant $R = 8.3\,\text{J/molK}$; the molar mass of CO_2, $\mu = 44\,\text{g/mol}$.

c) In Fig. 3.2.was plotted the atmospheric pressure values (in arbitrary units) as a function of time of descent forthe planet P_2atmosphere. When the probe touched the surface of the planet P_2, it sends the value of the temperature $T_0 = 750\,\text{K}$ and respectively the value of gravitational acceleration $g_0 = 8\,\text{ms}^{-2}$

Draw the following dependency graphs for $p = f(h)$ and $T = f(h)$ in the CO_2 atmosphere of the planet P_2.

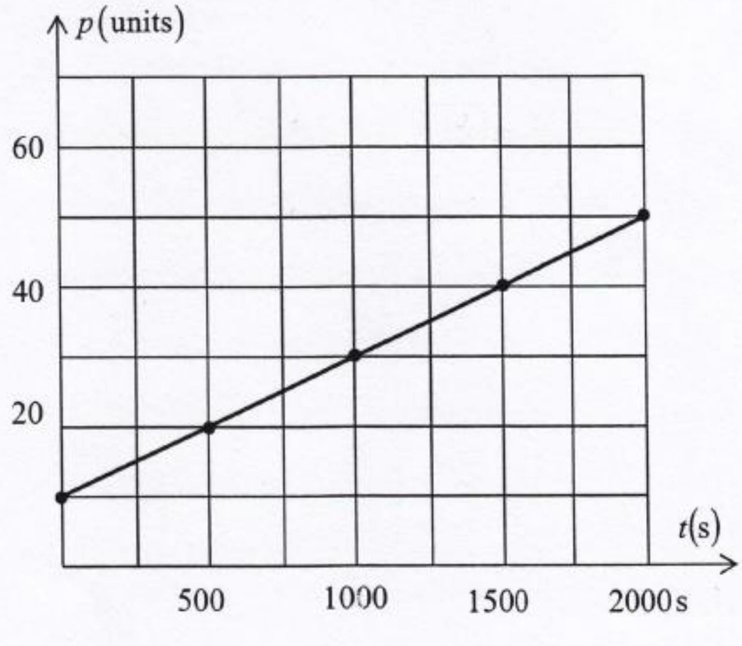


Fig. 3.2.

Problem 3 IOAA Observer on an extrasolar planet

The Sirius star, located in the constellation ofCanis Major, is the brightest star in the night sky of the Earth. What the observer's eye sees as a single star is actually a binary star system.

The high brightness of Sirius is a consequence of two facts: its intrinsic luminosity and its proximity to the Earth.

The Mizar multiple star system, in the constellation of Ursa Major, consists of 4 stars seen along the same line of sight from the Earth. Some of these stars form a gravitationally bound system.

Let's assume that an observer (observer A) is located on one of the planets of the Sirius system.

Determine:

a) The magnitude of the Sun as seen by observer A($m_{\text{Sun,Planet}}$).

b) The magnitude of Sirius star system as seen by the observer A.($m_{SY,\text{Planet}}$)

c) Thecombined intrinsic luminosity of theMizarsystem , L_{Mizar};

d) the average distance between gravitationally bound stars of the Mizar system and Earth, `

e) The geocentric angular distancebetweenMizarsystemand Sirius, $\Delta\theta$;

f) The physical distance between the gravitationally bound stars of theMizar system and the observer A. ($d_{\text{Mizar,Planet}}$)

g) The magnitude of the entire Mizar system as seen by the observer A.($m_{\text{Mizar,Planet}}$)

Also estimate amount of errorsin all your answers.

The following data may be used:

$d_{\text{Sirius},Earth} = 2.6$ pc - the Sirius – Earth distance;

$m_{\text{Sirius},Earth} = -1.46^{\text{m}}$ - the apparent magnitude of Sirius measured from the Earth;

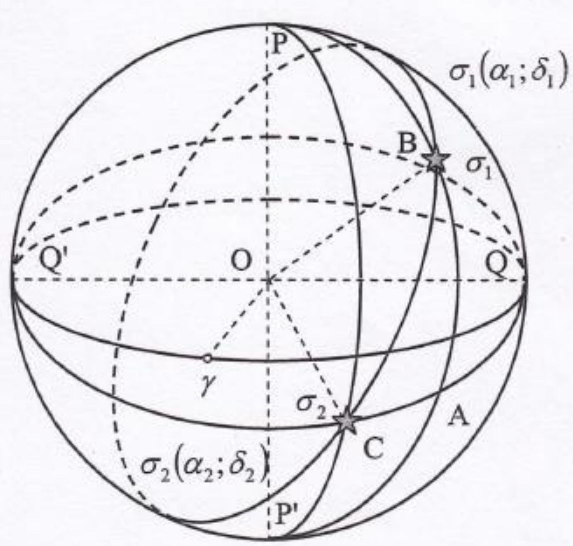
$d_{Sun,Earth} = 1$ AU - theSun – Earth distance;

$m_{\text{Sun},Earth} = -26{,}78^{\text{m}}$ - the apparent magnitude of the Sun as seen from Earth ;

$d_{\text{Sirius,Planet}} = 10$ AU - distance between Sirius and its planet where the observer A is located;

In the table below information for the stars from the Mizar system as measured from the Earth is given.

Star number	Name of the star	Apparent magnitude	Parallax (mili arc seconds)
1	Alcor	3.99 ± 0.01	39.91 ± 0.13
2	Mizar A	2.23 ± 0.01	38.01 ± 1.71
3	Mizar B	3.86 ± 0.01	38.01 ± 1.71
4	Sidus Ludoviciana	7.56 ± 0.01	8 ± 4



The equatorial coordinates of Mizar system (σ_1) and respectively of Sirius (σ_2) , located on the heliocentric map are :

$\alpha_{Mizar} = \alpha_1 = 13^{\text{h}}23^{\text{min}}55.5^{\text{s}}$;

$\delta_{Mizar} = \delta_1 = 54^\circ 55' 31''$; $\alpha_{\text{Sirius}} = \alpha_2 = 6^{\text{h}}45^{\text{min}}$;

$\delta_{\text{Sirius}} = \delta_2 = -16^\circ 43'$.

Note

$\ln(1-x) \approx -x$for $x \ll 1$

$e^x \approx 1 + x$ for $x \ll 1$